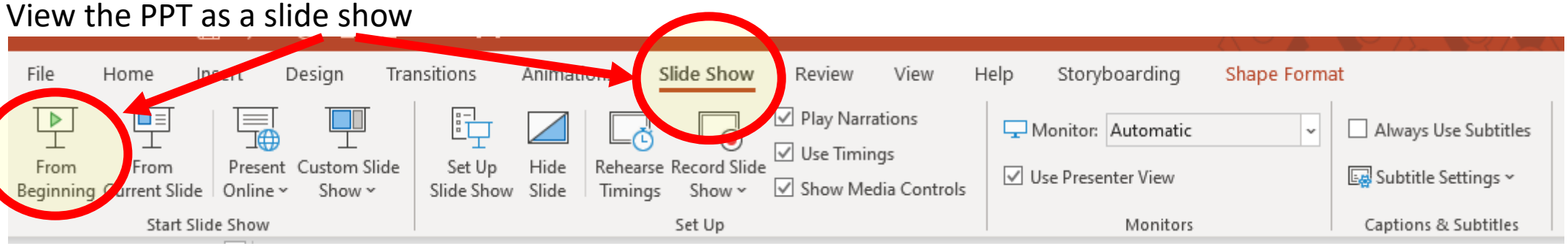
How to best use these slides…

- View the PPT as a slide show



- Then click through every step
  - Mouse clicks will advance the slide show
  - Left/right arrow keys move forward/backward
  - Mouse wheel scrolling moves forward/backward
- When a question is posed, stop and think it through, try to answer it yourself before clicking
- If you have questions, use PS discussion boards, email me, and/or visit us in a Teams class session!

LESSON 7.4b

Adding and Subtracting Rational Expressions with Unlike Denominators

Today you will:

- Add and subtract rational expressions that have unlike denominators
- Practice using English to describe math processes and equations

Core Vocabulary:

- Rational expression, p. 376
- LCD (Lowest Common Denominator)

Prior:

- Fractions and fraction arithmetic
- Polynomials
- LCM (Lowest Common Multiple) … yeah LCD is just LCM for fractions …

Review/Recap

- To add or subtract rational expressions you ***MUST*** have common denominators.
  - Add (or subtract) the numerators
  - Keep the denominator
  - Always simplify
- If the rational expressions have different denominators:
  - You need to make them the same … but without changing the fractions
  - Do this by finding the LCD (lowest common denominator)
  - …which is another way of saying “find the LCM (lowest common multiple) of the denominators
- To find the LCM of two expressions:
  1. Factor each expression completely
  2. Determine what is missing from each
  3. The LCM will have one of each factor:
     1. Combine the missing parts for each
     2. Make sure both are the same

So how exactly do we use LCM to add rational expressions?

First, here is an updated set of steps for adding:

1. Use LCM to get common denominators if necessary
2. Add (or subtract) the numerators
3. Keep the denominator
4. Always simplify

Let's start off simple: $\frac{3}{8} + \frac{5}{6}$

- Use the LCM process to find what is missing so we can make the denominators the same:
  - Prime factors of 8: 2 x 2 x 2 …missing 3
  - Prime factors of 6: 2 x 3 …missing 2 x 2 = 4
- Multiply each fraction by a new fraction "what's missing" over "what's missing"
  - $\frac{3}{8} \cdot \frac{3}{3}$ and $\frac{5}{6} \cdot \frac{4}{4}$ which will give us the LCD (lowest common denominator) of 24!
- Now do the "normal" thing:
  - Add the numerators, keep the denominator!

$$\frac{3}{8} + \frac{5}{6} = \frac{3}{8} \cdot \frac{3}{3} + \frac{5}{6} \cdot \frac{4}{4} = \frac{9}{24} + \frac{20}{24} = \frac{29}{24}$$

Let's make sure we got this …

Find the sum $\frac{x}{x^2-x-12}+\frac{5}{12x-48}$

1. Use the LCM process to find what is missing so we can get the LCD (make the denominators the same):
   - $x^2-x-12=(x+3)(x-4)$ missing the 12
   - $12x-48=12(x-4)$ missing the $(x+3)$
2. Get common denominators:
   - multiply each by 1 using the missing pieces(s) for both the numerator and denominator:

$$\frac{x}{x^2-x-12}+\frac{5}{12x-48}=\frac{x}{(x+3)(x-4)}+\frac{5}{12(x-4)}$$

Prime factors for each denominator

$$=\frac{x}{(x+3)(x-4)}\cdot\frac{12}{12}+\frac{5}{12(x-4)}\cdot\frac{(x+3)}{(x+3)}$$

Build LCD: multiplying by 1

…and simplify

$$=\frac{12x}{12(x+3)(x-4)}+\frac{5(x+3)}{12(x+3)(x-4)}=\frac{12x+5(x+3)}{12(x+3)(x-4)}=\frac{17x+15}{12(x+3)(x-4)}$$

Find the sum $\frac{7}{9x^2} + \frac{x}{3x^2 + 3x}$.

Find the LCD and then add. To find the LCD, factor each denominator and write each factor to the highest power that appears in either denominator.
Note that $9x^2 = 3^2x^2$ and $3x^2 + 3x = 3x(x + 1)$, so the LCD is $9x^2(x + 1)$

$$\frac{7}{9x^2} + \frac{x}{3x^2 + 3x} = \frac{7}{9x^2} + \frac{x}{3x(x + 1)}$$

Factor second denominator.

$$= \frac{7}{9x^2} \cdot \frac{x + 1}{x + 1} + \frac{x}{3x(x + 1)} \cdot \frac{3x}{3x}$$

LCD is $9x^2(x + 1)$.

$$= \frac{7x + 7}{9x^2(x + 1)} + \frac{3x^2}{9x^2(x + 1)}$$

Multiply.

$$= \frac{3x^2 + 7x + 7}{9x^2(x + 1)}$$

Add numerators.

Find the difference $\frac{x+2}{2x-2} - \frac{-2x-1}{x^2-4x+3}$.

SOLUTION

$$\frac{x+2}{2x-2} - \frac{-2x-1}{x^2-4x+3} = \frac{x+2}{2(x-1)} - \frac{-2x-1}{(x-1)(x-3)}$$

Factor each denominator.

$$= \frac{x+2}{2(x-1)} \cdot \frac{x-3}{x-3} - \frac{-2x-1}{(x-1)(x-3)} \cdot \frac{2}{2}$$

LCD is $2(x-1)(x-3)$.

$$= \frac{x^2-x-6}{2(x-1)(x-3)} - \frac{-4x-2}{2(x-1)(x-3)}$$

Multiply.

$$= \frac{x^2-x-6-(-4x-2)}{2(x-1)(x-3)}$$

Subtract numerators.

$$= \frac{x^2+3x-4}{2(x-1)(x-3)}$$

Simplify numerator.

$$= \frac{(x-1)(x+4)}{2(x-1)(x-3)}$$

Factor numerator. Divide out common factors.

$$= \frac{x+4}{2(x-3)}, \; x \neq 1$$

Simplify.

COMMON ERROR

When subtracting rational expressions, remember to distribute the negative sign to all the terms in the quantity that is being subtracted.

Review/Recap

- To add or subtract rational expressions you ***MUST*** have common denominators.
    1. Use LCM to get common denominators (LCD) if necessary
    2. Add (or subtract) the numerators
    3. Keep the denominator
    4. Always simplify
- To find the LCM of two expressions:
    1. Factor each expression completely
    2. Determine what is missing from each
    3. The LCM will have one of each factor:
        1. Combine the missing parts for each
        2. Make sure both are the same

Homework

Pg 388, #17-26